Closure of Shallow Water type equations for various Mixing Length turbulent models

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ABSTRACT

We propose a systematic and general methodology to derive two moments approximate models of shallow water type for thin films down an incline. Some recent related works are [2, 3]. We treat here the case of a turbulent flow, modeled by a Mixing Length law of strain rate (see [1] for non newtonian fluids and [4] for laminar flows). The key point of the method is precisely the inversion of this law, that we want to be done in the most general way. We perform a long wave asymptotic of the Navier-Stokes equations and construct a Hilbert expansion up to the second order with respect to the small aspect ratio (or film parameter) ε of the solution. This expansion is singular because of the turbulence model. Therefore we perform new approximations in order to catch the viscous boundary layer near the bottom. Error estimates in terms of another small parameter η are then given. Finally we are able to construct a whole family of consistent shallow water models, taking into account the turbulence.

References

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